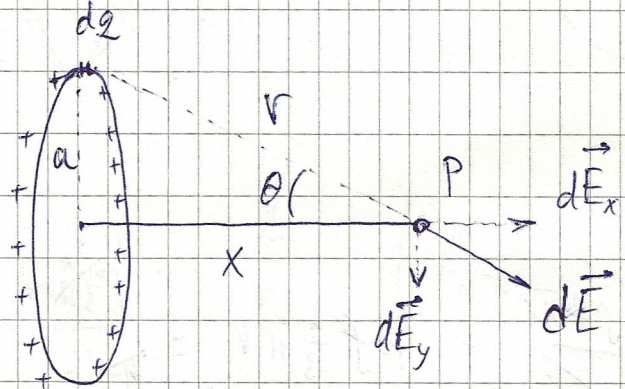
ELECTRIC FIELD OF UNIFORM RING OF CHARGE

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.



$$d\vec{E} = k\frac{dq}{r^2} \quad ; \quad r = \sqrt{x^2+a^2} \quad ; \quad \cos\theta = \frac{x}{r}$$

$$E_y = \int dE\sin\theta = 0$$

Y-COMPONENTS OF TOTAL ELECTRIC FIELD dE WILL CANCEL OUT EACHOTHER DUE TO SYMMETRY.

$$E_x = \int dE\cos\theta = \int k\frac{dq}{r^2}\frac{x}{r} = \frac{kx}{\sqrt{(x^2+a^2)^3}}\int dq = \boxed{\frac{kxQ}{\sqrt{(x^2+a^2)^3}}}$$

NOTE THAT IF $x \ll a$ THEN

$$(x^2+a^2)^{\frac{3}{2}} = (a^2)^{\frac{3}{2}}\left(\frac{x^2}{a^2}+1\right)^{\frac{3}{2}} = a^3 \qquad \left(\frac{x^2}{a^2} \approx 0\right)$$

THEREFORE MOTION OF THE CHARGE INSIDE A CHARGED RING IS SIMPLE HARMONIC FOR SMALL DISPLACEMENTS ($x \ll a$).